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On the solutions of the NeutroQuadrupleRing of polynomials

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Abstract: In this paper, we give some basic and elementary properties of the Neutron Quadruple Ring of polynomials and finally make some analysis as regards the NeutronQuadrupleRing of polynomials of the first degree.

Keywords: *Neutron Quadruple Ring of polynomials, Neutron Quadruple number NQ(x), single indeterminate, neutrosophic ring, neutron Quadruple field NQ(F)*

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1 Introduction

The necessary and sufficient conditions for (NQ(x),+, .) to be a Neutron Quadruple Ring was given in [1]. In this work, we will give some basic definitions, examples and results involving neutron Quadruple Ring of polynomials.

Definition

Suppose that (NQ(x),+, .) is a neutrosophic ring and x, an indeterminate.

An infinite formal sum given by : NQ(x) = $\sum_{i=0}^{\infty}(a_1, a_2T, a_3I, a_4F)_i x^i$

(where each $a_k \in \mathbb{R}$ or $\mathbb{C}$ ∀ k = 1,2, . . . ,4) is known as a formal power series in x. Here, each of the $(a_1, a_2T, a_3I, a_4F)_i$ is a coefficient in the neutron Quadruple number NQ(x) for which the $a_k \in \mathbb{R}$ or $\mathbb{C}$ ∀ k. Now, suppose that NQ[[x]] denotes the set of all such power series. Defne "⊕" to be the addition and "⊙" **the multiplication in** NQ[[x]] by $N_aQ[[x]] \oplus N_bQ[[x]]$ = ? and $N_aQ[[x]] \odot N_bQ[[x]]$ = ? , where $N_aQ[[x]]$, $N_bQ[[x]] \in NQ[[x]]$. And we call the triple (NQ[[x]] , ⊕ , ⊙) **the neutroQuadrupleRing of formal power series in the single indeterminate x with coefficients in** NQ[[x]].

Remarks

If NQ[x] is commutative, s is NQ[[x]]. Also, NQ[[x]] has identity iff NQ[x] does.

Now, if it is required that in the infinite formal sum for each NtQ(x) as defined, al except a finite number of the coefficients are zero, then NQ(x) is called a neutroQuadruple polynomial in the single indeterminate x. If (a_1,a_2 T,a_3 I,a_4 F)_i = 0 for all >n , and (a_1,a_2 T,a_3 I,a_4 F)_n ≠ 0, then, NaQ(x) is a neutroQuadruple polynomial of degree n and (a_1,a_2 T,a_3 I,a_4 F)_n is said to be the leading coefficient of the NaQ(x). We then write NaQ(x) = ∑_(i=0)^n▒〖(a_1,a_2 T,a_3 I,a_4 F)_i x^i 〗, which implies that NaQ(x) = (a_1,a_2 T,a_3 I,a_4 F)_0 + (a_1,a_2 T,a_3 I,a_4 F)_1 x + (a_1,a_2 T,a_3 I,a_4 F)_2 x^2 + . . . + (a_1,a_2 T,a_3 I,a_4 F)_(n-1) x^(n-1) + (a_1,a_2 T,a_3 I,a_4 F)_n x^n. Usually, we refer to (a_1,a_2 T,a_3 I,a_4 F)_0 as the constant term of the neutron Quadruple NaQ(x) under the definitions of the addition and multiplication given above for the power series. The neutron Quadruple set NQ[x] of polynomials in the indeterminate x over the neutron Quadruple field NQ(F) can also be seen as a neutrosophicRing.

Definition (see [1])

A neutrosophic quadruple number is a number of the form (a, bT, cI, dF) where (T, I, F) have their usual neutrosophic logic meanings and a, b, c, d ∈ R or C. The set NQ defined by:

NQ = {(a, bT, cI, dF): a, b, c, d ∈ R or C.}

By definition 1.1, we have the emergence of the neutronQuadruplePolynomial Ring as follows:

Definition

A neutron Quadruple Ring of polynomials is of the form given by :

NQ[x] = $\sum_{k=0}^{n} C_k x^k$ for a single indeterminate x,

where C_k = (a_k, b_kT, c_kI, d_kF): a, b, c, d ∈ $\mathbb{R}$ or $\mathbb{C}$.

Hence, the operations of addition, subtraction, multipliation and division can thus be formulated for the neuroQuadruplePolynomialRing.

Addition

Given that : **NQ[x](1)** $= \sum_{k=0}^{m}(\mathbf{a_k, b_kT, c_kI, d_kF})x^k$ **and**

$$\mathbf{NQ[x](2)} = \sum_{k=0}^{n}(\mathbf{e_k, f_kT, g_kI, h_kF})x^k$$

Then, NQ[x](1) + NQ[x](2) $= \sum_{k=0}^{\max\{m,n\}}(A_k, B_kT, C_kI, D_kF)x^k$, where $A_k = (a_k + e_k)$, $B_k = (b_k + f_k)$, $C_k = (c_k + g_k)$, and $D_k = (d_k + h_k)$. The operation of subtraction is conducted analogously**.**

Multiplication

NQ[x](1).NQ[x](2) = $\sum_{k=0}^{m+n}(A_k, B_kT, C_kI, D_kF)x^k$, where $A_I = \sum_{i=0,j=k-i}^{k} a_i e_j$, $B_i = \sum_{i=0,j=k-i}^{k} b_i f_j$, $C_i = \sum_{i=0,j=k-i}^{k} c_i g_j$, and $D_i = \sum_{i=0,j=k-i}^{k} d_i h_j$.

2 Division algorithm

The major rule for any division algorithm to hold or being valid is that for such (2) to divide (2), the degree m must be greater than n. Hence, for NQ[x](2) to divide NQ[x](1) , there exists

NQ[x](3)and NQ[x](4) such that NQ[x](1) = NQ[x](2).NQ[x](3) + NQ[x](4) and deg{NQ[x](4)} < deg{NQ[x](2)} or NQ[x](4) = 0.

3 Linear neutroQuadrupleRing of polynomials

$(b_1, b_2T, b_3I, b_4F).X = (a_1, a_2T, a_3I, a_4F).$

Let X = $\frac{(a_1,a_2T,a_3I,a_4F)}{((b_1,b_2T,b_3I,b_4F))} = (c_1, c_2T, c_3I, c_4F)$ say,

We have that :

$(a_1, a_2T, a_3I, a_4F) = (b_1, b_2T, b_3I, b_4F).(c_1, c_2T, c_3I, c_4F)$

From this , the following relationships may be obtained :

$a_1 = b_1c_1$ . . . .(1)

$a_2 = b_1c_2 + (c_1 + c_2)b_2$ . . . .(2)

$a_3 = b_1c_3 + b_2c_3 + (c_1 + c_2 + c_3)b_3$ . . . . (3)

$a_4 = (b_1 + b_2 + b_3)c_4 + (c_1 + c_2 + c_3 + c_4)b_4$ . . . .(4)

where we have

$b_1 \mid a_1$

$(b_1+b_2) \mid (a_1 + a_2)$ i.e. $\sum_{i=1}^{2} b_i \mid \sum_{i=1}^{2} a_i$

$(b_1+b_2 + b_3) \mid (a_1 + a_2 + a_3)$ i.e. $\sum_{i=1}^{3} b_i \mid \sum_{i=1}^{3} a_i$ and

$(b_1+b_2 + b_3 + b_4) \mid (a_1 + a_2 + a_3 + a_4)$ i.e. $\sum_{i=1}^{4} b_i \mid \sum_{i=1}^{4} a_i$

In general, I could be proposed that : $\sum_{i=1}^{n} b_i \mid \sum_{i=1}^{n} a_i$ for every integer n

Based on this observation, some conditions for divisibility can be as well proposed which could be further useful in the general divisibility concepts of the neutroQuadrupleRing of polynomials.

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